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Locally Most Powerful Group-Sequential Tests with Groups of Observations of Random Size: Finite Horizon

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Abstract—We consider sequential hypothesis testing based on observations which are received in groups of random size. The observations are supposed independent both within and between the groups, with a distribution depending on a real-valued parameter θ. We suppose that the group sizes are independent and their distributions are known, and that the groups are formed independently from the observations. We are concerned with a problem of testing a simple hypothesis $H_0 : \theta = \theta_0$ against a composite alternative $H_1 : \theta > \theta_0$, supposing that no more than a given number of groups will be available (finite horizon). For any (group-)sequential test, we take into account the following three characteristics: its error probability of the first type, the derivative of its power function at $\theta = \theta_0$, and the average cost of observations, under some natural assumptions about the cost structure. Under suitable regularity conditions, we characterize the structure of all sequential tests maximizing the derivative of the power function among all (finite-horizon) sequential tests whose error probability of the first type and the average cost of observations do not exceed some prescribed levels.

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1. INTRODUCTION

In this article, we consider sequential hypothesis testing when the observations are received in groups of a random size, rather than on a one-at-a-time basis. We adhere to the statistical model proposed by N. Mukhopadhyay and B.M. de Silva [2] for testing a simple hypothesis against a simple alternative. There are many practical situations where the random group size model comes into question and a lot of theoretical problems that arise (see [2]). Leaving the theoretical analysis of the case of two simple hypotheses till another publication, we address in this article the problem of testing a simple hypothesis against a composite (one-sided) alternative.

In this article, we follow the "local" approach by R.H. Berk [1] based on maximizing the derivative of the power function, at the point of the null hypothesis. Berk [1] calls locally most powerful the tests that maximize the derivative of the power function among all the tests whose error probability of the first type and the average sample number do not exceed some prescribed levels. N. Schmitz [8] extends this approach to the scheme of sequentially planned experiments. In [3], we study the locally most powerful tests for a more general case of dependent observations, and in [4] we apply these results to the case of independent observations. In this article, we use the same approach for the case of random groups of independent and identically distributed (i.i.d.) observations.

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